Analytical Solution of Two-Body System with Yamaguchi Potential

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Abstract

The analytical solution of the bound and scattering quantum states is derived based on Lippmann-Schwinger equation which can also be derived from well known Schrödinger equation. From the transition matrix element in momentum space, the phase shift which is one of the scattering parameters is calculated and the nature of interaction is deduced from the phase shift results. A two-body bound state with Yamaguchi potential is determined analytically by solving Lippmann-Schwinger equation. The binding energy of deuteron is evaluated analytically and it is 2.22 MeV.

Key words: bound and scattering states, Lippmann-Schwinger equation, T- Matrix, phase shift.

1. Introduction

Almost everything we know about nuclear and atomic physics has been discovered by scattering experiments, e.g. Rutherford's discovery of the nucleus, the discovery of sub-atomic particles (such as quarks), etc. The nature of interaction between particles can be revealed by studying the scattering parameters such as phase shift, scattering length, effective range and cross-section. These parameters can easily be deduced from transition matrix. The transition matrix is primitive concept of scattering matrix. The study of scattering process is very important for the students who are studying at undergraduate and postgraduate level. A two-body bound state with Yamaguchi potential is determined analytically by solving Lippmann-Schwinger equation.In this paper, the very simple and clear formulation of the scattering and bound states was formulated by using Yamaguchi potential.

2. Formalism

Derivation of Lippmann-Schwinger Equation From Schrödinger Equation

The time-independent form of the Schrödinger equation can be used to describe scattering processes. The Schrödinger equation can be written as

$$\left(\hat{H}_0 + \hat{V}\right)|\Psi\rangle = E|\Psi\rangle \tag{1.1}$$

where H_0 stands for the kinetic energy operator, $\hat{H}_0 = \hat{p}^2/2\mu$ in momentum space and $\hat{H}_0 = -\vec{\nabla}^2/2\mu$ in configuration space. The momentum eigenstate of $\hat{H}_0$ is defined by

$$\hat{H}_0|p\rangle = \frac{p^2}{2\mu}|p\rangle \tag{1.2}$$

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$$\hat{\mathrm{p}}|\,p\rangle = p\ |\,p\rangle \quad (1.3)$$

The potential that scatters a particle coming in is assumed to be short ranged, i.e. V= 0 beyond a certain distance "a" where "a" is the size of the scatterer. In this condition there exists only the free motion of the particle and it is represented by the free particle momentum eigenstate $|\mathbf{p}\rangle$. We will use the general notation of this free particle momentum eigenstate $|\mathbf{p}\rangle$ as $|\psi_0\rangle$ and Eq.(1.2) can be reduced to free motion

$$\hat{\mathrm{H}}_0\,|\Psi_0\rangle = \frac{\mathrm{p}^2}{2\mu}|\psi_0\rangle = \mathrm{E}|\Psi_0\rangle \quad (1.4)$$

We are interested in the scattering process caused by the effect of the potential "V". Because of this potential effect, the energy eigenstate differs from the free particle state. If it is an elastic scattering, there will be no change in energy. To find out about the effect of the potential, we reorder the Eq.(1.1).

$$(\mathrm{E}-\hat{\mathrm{H}}_0)|\Psi\rangle = \hat{\mathrm{V}}|\Psi\rangle \quad (1.5)$$

Naively we can write the solution of Eq.(1.5) as follows:

$$|\Psi\rangle = \frac{1}{\mathrm{E}-\hat{\mathrm{H}}_0}\hat{\mathrm{V}}|\Psi\rangle \quad (1.6)$$

When we go to the scattering process, we will apply the following Lippmann-Schwinger equation. These will be singularity when the value of H_0 becomes close to E. To avoid the singularity of the operator $\frac{1}{\mathrm{E}-\hat{\mathrm{H}}_0}$, energy E is made to be slightly complex value. We can write the solution of Eq.(1.6) as

$$|\Psi^{(+)}\rangle = |\Psi_0\rangle + \frac{1}{\mathrm{E}+\mathrm{i}\varepsilon-\hat{\mathrm{H}}_0}\hat{\mathrm{V}}|\Psi^{(+)}\rangle \quad (1.7)$$

where, $(\hat{\mathrm{H}}_0-\mathrm{E})|\Psi_0\rangle = 0$.

This is the Lippmann-Schwinger equation in ket form. The state $|\Psi^{(+)}\rangle$ is the scattering state for an outgoing wave in momentum space generated by the potential operator V. The iε is the correct boundary condition for an outgoing wave and $|\Psi_0\rangle$ is the free momentum state which initiates the scattering process.

Let $|\Psi_0\rangle = |\mathbf{p_0}\rangle$ and $|\Psi^{(+)}\rangle = |\Psi^{(+)}_{\mathbf{p_0}}\rangle$. By taking the inner product of Eq. (1.7) with the bra $\langle\mathbf{p}|$, we get

$$\langle\mathbf{p}|\Psi^{(+)}_{\mathbf{p_0}}\rangle = \langle\mathbf{p}|\mathbf{p_0}\rangle + \langle\mathbf{p}|\frac{\hat{\mathrm{V}}}{\mathrm{E}+\mathrm{i}\varepsilon-\hat{\mathrm{H}}_0}|\Psi^{(+)}_{\mathbf{p_0}}\rangle \quad (1.8)$$

By operation of $\hat{\mathrm{H}}_0$ on the bra $\langle\mathbf{p}|$, we can express the equation in simple form.

$$\Psi^{(+)}_{p_0}(\mathbf{p})=\delta(\mathbf{p}-\mathbf{p_0})+\frac{1}{E+i\varepsilon-\frac{p^2}{2\mu}}\langle\mathbf{p}|\hat{V}|\Psi^{(+)}_{p_0}\rangle \tag{1.9}$$

where, $\langle\mathbf{p}|\Psi^{(+)}_{p_0}\rangle=\Psi^{(+)}_{p_0}(\mathbf{p})$ and $\langle\mathbf{p}|\mathbf{p_0}\rangle=\delta(\mathbf{p}-\mathbf{p_0})$. We insert the completeness relation $\int d\mathbf{p}'|\mathbf{p}'\rangle\langle\mathbf{p}'|=1$ into the above equation and becomes

$$\Psi^{(+)}_{p_0}(\mathbf{p})=\delta(\mathbf{p}-\mathbf{p_0})+\frac{1}{E+i\varepsilon-\frac{p^2}{2\mu}}\int d\mathbf{p}'\langle\mathbf{p}|\hat{V}|\mathbf{p}'\rangle\Psi^{(+)}_{p_0}(\mathbf{p}') \tag{1.10}$$

Transition Matrix

The above equation contains the driving term in the form of a delta function and the pole term. We define a new quantity as

$$\langle\mathbf{p}|\hat{V}|\Psi^{(+)}_{p_0}\rangle=T(\mathbf{p},\mathbf{p_0}) \tag{2.1}$$

We rewrite the Eq.(1.10) by inserting the operator $\hat{V}$ into both sides and the completeness relation into the right hand side of it. Then Eq.(1.10) becomes

$$\langle\mathbf{p}|\hat{V}|\Psi^{(+)}_{p_0}\rangle=\langle\mathbf{p}|\hat{V}|\mathbf{p_0}\rangle+\int d\mathbf{p}'\langle\mathbf{p}|\hat{V}|\mathbf{p}'\rangle\frac{1}{E+i\varepsilon-\frac{p'^2}{2\mu}}\langle\mathbf{p}'|\hat{V}|\Psi^{(+)}_{p_0}\rangle \tag{2.2}$$

We can write down Eq.(2.2) with the help of Eq.(2.1) as

$$T(\mathbf{p},\mathbf{p_0})=V(\mathbf{p},\mathbf{p_0})+\int d\mathbf{p}'\ V(\mathbf{p},\mathbf{p}')\ \frac{1}{E+i\varepsilon-\frac{p'^2}{2\mu}}\ T(\mathbf{p}',\mathbf{p_0}) \tag{2.3}$$

where, $T(\mathbf{p},\mathbf{p_0})$ is transition matrix (T-matrix), which interprets as the transition from the initial momentum $\mathbf{p_0}$ to the final momentum $\mathbf{p}$ because of the driving term $V(\mathbf{p},\mathbf{p_0})$. Therefore, T-matrix elements play an important role because they carry physical information of considering system.

Calculation of T-matrix Element in Momentum Space

In this section, we will discuss the two body T-matrix elements. For the scattering process the Lipmann-Schwinger Equation for T-matrix in operator form is

$$\hat{T}=\hat{V}+\hat{V}\,G_0\,\hat{T} \tag{3.1}$$

where, G_0 is free propagator, $G_0=1/\left(E_0-\hat{p}^2/2\mu\right)$. We need to project onto momentum space as

$$\langle p|\hat{T}|p'\rangle=\langle p|\hat{V}|p'\rangle+\langle p|\hat{V}G_0\,\hat{T}|p'\rangle \tag{3.2}$$

We insert the completeness relation onto the second part of the Eq. (3.2), then

$$\langle p|\hat{T}|p'\rangle = \langle p|\hat{V}|p'\rangle + \int p''^2\, dp'' \langle p|\hat{V}G_0|p''\rangle\langle p''|\hat{T}|p'\rangle \tag{3.3}$$

By operation of G_0 on $|p''\rangle$ we get

$$\langle p|\hat{T}|p'\rangle = \langle p|\hat{V}|p'\rangle + \int p''^2\, dp'' \frac{1}{E_0 - \frac{p''^2}{2\mu}} \langle p|\hat{V}|p''\rangle\langle p''|\hat{T}|p'\rangle \tag{3.4}$$

where, E_0 is incident energy and $E_0 = p_0^2/2\mu$, the Eq. (3.4) becomes

$$\langle p|\hat{T}|p'\rangle = \langle p|\hat{V}|p'\rangle + 2\mu \int p''^2\, dp'' \frac{1}{p_0^2 - p''^2} \langle p|\hat{V}|p''\rangle\langle p''|\hat{T}|p'\rangle \tag{3.5}$$

It can encounter the situation $p_0 = p''$, then the integral will diverge and so we add $i\varepsilon$ to overcome this divergence.

$$\langle p|\hat{T}|p'\rangle = \langle p|\hat{V}|p'\rangle + 2\mu \int p''^2\, dp'' \frac{1}{p_0^2 + i\varepsilon - p''^2} \langle p|\hat{V}|p''\rangle\langle p''|\hat{T}|p'\rangle \tag{3.6}$$

This is the same as Eq.(2.3). This equation can be solved directly by introducing a small value of $i\varepsilon$, this technique will appear on the upcoming volume of this journal.

We emphasize the second part of Eq. (3.6) and adding and subtracting the additional term $2\mu \int dp'' \frac{p_0^2}{p_0^2 + i\varepsilon - p''^2} \langle p|V|p_0\rangle\langle p_0|T|p_0\rangle$ and regroup this equation. And then we apply the principal value theorem, which is defined as

$$\lim_{\varepsilon \to 0} \frac{q}{x' + i\varepsilon} = \frac{q}{x'} - i\pi\delta(x')$$

Then we use the property of delta function $\delta(x^2 - a^2) = \frac{1}{2|a|}[\delta(x+a) + \delta(x-a)]$ and the standard integration form $\int_0^\infty \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \frac{x-a}{x+a}$. We can get

$$\begin{aligned}\langle p|\hat{T}|p'\rangle = \langle p|\hat{V}|p'\rangle &+ 2\mu \int dp'' \frac{p''^2 \langle p|V|p''\rangle\langle p''|T|p'\rangle - p_0^2 \langle p|V|p_0\rangle\langle p_0|T|p_0\rangle}{p_0^2 - p''^2} \\ &+ \mu p_0 \langle p|V|p_0\rangle\langle p_0|T|p_0\rangle \left[\ln\left|\frac{p_0 + p_{max}}{p_{max} - p_0}\right| - i\pi \right]\end{aligned} \tag{3.7}$$

Our integral limit is from zero to infinity but we determine that p_{max} is enough for that limit. For $p' = p_0$, the Eq. (3.7) is

$$\langle p|T|p_0\rangle = \langle p|V|p_0\rangle + 2\mu \int dp'' \frac{p''^2 \langle p|V|p''\rangle\langle p''|T|p_0\rangle - p_0^2 \langle p|V|p_0\rangle\langle p_0|T|p_0\rangle}{p_0^2 - p''^2}$$

$$+\mu p_0 \langle p|V|p_0\rangle\langle p_0|T|p_0\rangle\left[\ln\left|\frac{p_0+p_{max}}{p_{max}-p_0}\right|-i\pi\right] \tag{3.8}$$

The above equation is integral form of T-matrix elements and it is usually used to study the scattering problem.

Analytical Solution of T-matrix

To calculate the analytical solution of T-matrix we use the separable Yamaguchi potential. The S-wave potential is of the form

$$V(p,p')=\lambda\, g(p)\, g(p') \tag{4.1}$$

where, g’s are the form factors and which are the function of momentum

$$g(p)=\frac{1}{\beta^2+p^2} \tag{4.2}$$

and p’s are the initial and final momenta between the two nucleons, respectively. The parameters of the Yamaguchi potential for $^1S_0$ state are $\lambda = -0.5592$ MeV fm^{-1} and β=1.13 fm^{-1}. We take the T-matrix is of the following form

$$T(p,p')=\phi g(p) g(p'). \tag{4.3}$$

We insert this equation in Eq.(3.8) and the intermediate step is

$$\phi g(p) g(p')=\lambda g(p) g(p')+2\mu\phi\lambda g(p) g(p')\int p''^2 dp''\frac{1}{\left(p_0^2-p''^2\right)} g^2(p''). \tag{4.4}$$

We rearrange Eq.(4.4) and the intermediate step of constant ϕ as

$$\phi = \frac{\lambda}{1-2\mu\lambda\int p''^2 dp''\frac{1}{\left(p_0^2-p''^2\right)}\; g^2(p'')}. \tag{4.5}$$

Eq.(4.3) becomes

$$T(p,p')=\frac{\lambda}{1-2\mu\lambda\int p''^2\, dp''\frac{1}{\left(p_0^2-p''^2\right)} g^2(p'')} g(p)\, g(p'). \tag{4.6}$$

If we know ϕ, we can solve the Eq. (4.6). By using Eq.(4.2) and Eq.(4.5) becomes as

$$\phi = \frac{\lambda}{1-2\mu\lambda\int p''^2 dp''\frac{1}{\left(p_0^2-p''^2\right)}\frac{1}{\left(\beta^2+p''^2\right)^2}}. \tag{4.7}$$

We rearrange Eq.(4.7) as

$$\phi = \frac{\lambda}{1+2\mu\lambda\int p''^2 dp'' \frac{1}{\left(p''^2 - p_0{}^2\right)} \frac{1}{\left(p''^2+\beta^2\right)^2}} \tag{4.8}$$

We consider $\frac{1}{\left(p''^2+\beta^2\right)^2} = \frac{1}{-2\beta}\frac{\partial}{\partial\beta}\frac{1}{\left(p''^2+\beta^2\right)}$ and Eq.(4.8) becomes

$$\phi = \frac{\lambda}{1-2\mu\lambda\frac{\partial}{2\beta\partial\beta}\int p''^2 dp'' \frac{1}{\left(p''^2 - p_0{}^2\right)} \frac{1}{\left(p''^2+\beta^2\right)}} \tag{4.9}$$

We consider as a complex number and then

$$\phi = \frac{\lambda}{1-2\mu\lambda\frac{\partial}{2\beta\partial\beta}\int_0^\infty p''^2 dp'' \frac{1}{\left(p''^2 - p_0{}^2\right)} \frac{1}{\left(p''^2-(-i\beta)^2\right)}} \tag{4.10}$$

The pole appears at $p''=\pm p_0$ and $p''=\pm i\beta$, the integral will diverge. Therefore we use Residue theorem. We use, $\mathrm{Residue}(b) = \lim_{z\to z_0}(z-z_0)f(z)$ and then

$$\mathrm{Res}(+p_0) = \lim_{p''\to p_0}(p''-p_0)\frac{p''^2}{(p''-p_0)(p''+p_0)}\frac{1}{\left(p''^2+\beta^2\right)} \tag{4.11}$$

$$\mathrm{Res}(+p_0) = \frac{p_0}{2\left(p_0{}^2+\beta^2\right)} \tag{4.12}$$

$$\mathrm{Res}(+i\beta) = \lim_{p''\to i\beta}(p''-i\beta)\frac{p''^2}{\left(p''^2-p_0{}^2\right)(p''-i\beta)}\frac{1}{(p''+i\beta)} \tag{4.13}$$

$$\mathrm{Res}(+i\beta) = \frac{\beta}{2i\left(p_0{}^2+\beta^2\right)} \tag{4.14}$$

Residue Theorem, $\oint f(z)dz = 2\pi i\sum \mathrm{Res}$.

Let say the integral to be A

$$A = 2\pi i\left[\mathrm{Res}(+p_0)+\mathrm{Res}(+i\beta)\right] \tag{4.15}$$

$$A = 2\pi i\left(\frac{p_0}{2\left(p_0^2+\beta^2\right)}+\frac{\beta}{2i\left(p_0^2+\beta^2\right)}\right) \tag{4.16}$$

$$A = \pi\left(\frac{ip_0}{\left(p_0^2+\beta^2\right)}+\frac{\beta}{\left(p_0^2+\beta^2\right)}\right). \tag{4.17}$$

Now, we get the solution of Eq.(4.10) as

$$\phi = \frac{\lambda}{1 - 2\mu\lambda\pi \dfrac{\partial}{4\beta\partial\beta}\left(\dfrac{ip_0}{\left(p_0^2+\beta^2\right)} + \dfrac{\beta}{\left(p_0^2+\beta^2\right)}\right)}. \tag{4.18}$$

After differentiating Eq.(4.18),

$$\phi = \frac{\lambda}{1 - 2\mu\pi\lambda \dfrac{1}{4\beta}\left(\dfrac{-2i\beta p_0}{\left(p_0^2+\beta^2\right)^2} + \dfrac{1}{\left(p_0^2+\beta^2\right)} + \dfrac{-2\beta^2}{\left(p_0^2+\beta^2\right)^2}\right)} \tag{4.19}$$

Regrouping Eq.(4.19) and then

$$\phi = \frac{\lambda}{1 - 2\mu\pi\lambda \dfrac{1}{4\beta}\left(\dfrac{p_0^2 - 2i\beta p_0 - \beta^2}{\left(p_0^2+\beta^2\right)^2}\right)}, \tag{4.20}$$

Finally, we get the solution in simplest form as

$$\phi = \frac{\lambda}{1 - 2\mu\pi\lambda \dfrac{1}{4\beta}\left(\dfrac{\left(p_0 - i\beta\right)^2}{\left(p_0^2+\beta^2\right)^2}\right)}. \tag{4.21}$$

Now, we get analytical T matrix for single channel as

$$T\left(p, p'\right) = \frac{\lambda}{1 - 2\mu\pi\lambda \dfrac{1}{4\beta}\left(\dfrac{\left(p_0 - i\beta\right)^2}{\left({p_0}^2+\beta^2\right)^2}\right)} g\left(p\right) g\left(p'\right). \tag{4.22}$$

The Analytical Calculation for Two-Body Bound State

We will solve two-body bound state with Yamaguchi potential analytically. The Lippmann Schwinger equation for two-body system in single channel can be written as

$$\Psi\left(p\right) = \frac{1}{E - \dfrac{p^2}{\mu}} \int_0^\infty p'^2 dp' \lambda g\left(p\right) g\left(p'\right) \Psi\left(p'\right) \tag{5.1}$$

The above equation can be simplified as

$$\Psi\left(p\right) = \frac{1}{E - \dfrac{p^2}{\mu}} g\left(p\right) \lambda \int p'^2 dp' g\left(p'\right) \Psi\left(p'\right) \tag{5.2}$$

We set as,

$$C = \lambda \int_0^\infty p'^2 dp' g\left(p'\right) \Psi\left(p'\right) \tag{5.3}$$

The analytical solution of wave function can be written as,

$$\Psi(p) = C\frac{1}{E-\frac{p^2}{\mu}}g(p) \tag{5.4}$$

Now, we have to find the solution of Eq. (5.3) with the help of Eq. (5.4) and one can write easily as

$$C = \lambda\int_0^\infty p'^2dp'g(p')C\frac{1}{E-\frac{p'^2}{\mu}}g(p') \tag{5.5}$$

We set E as

$$E = -\frac{\alpha^2}{\mu} \tag{5.6}$$

Eq. (5.5) becomes

$$C = \lambda\int_0^\infty p'^2dp'g^2(p')C\frac{1}{-\frac{\alpha^2}{\mu}-\frac{p'^2}{\mu}} \tag{5.7}$$

$$C = -\lambda\mu C\int_0^\infty p'^2dp'g^2(p')\frac{1}{p'^2+\alpha^2} \tag{5.8}$$

$$C = -\lambda\mu C\int_0^\infty p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)^2}\frac{1}{\left(p'^2+\alpha^2\right)} \tag{5.9}$$

$$C = -\frac{1}{2}\lambda\mu C\int_{-\infty}^\infty p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)^2}\frac{1}{\left(p'^2+\alpha^2\right)} \tag{5.10}$$

where, $\frac{1}{\left(p'^2+\beta^2\right)^2} = \frac{1}{-2\beta}\frac{\partial}{\partial\beta}\frac{1}{\left(p'^2+\beta^2\right)}$

$$C = \frac{1}{4\beta}\lambda\mu C\frac{\partial}{\partial\beta}\int_{-\infty}^\infty p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)}\frac{1}{\left(p'^2+\alpha^2\right)} \tag{5.11}$$

$$C = \frac{1}{4\beta}\lambda\mu C\frac{\partial}{\partial\beta}\int_{-\infty}^\infty p'^2dp'\frac{1}{\left(p'^2-(-i\beta)^2\right)}\frac{1}{\left(p'^2-(-i\alpha)^2\right)} \tag{5.12}$$

The pole appear at $p' = \pm i\beta$ and $p' = \pm i\alpha$. Therefore we use Residue theorem.

$$\mathrm{Residue}(b) = \lim_{z\to z_0}(z-z_0)f(z) \tag{5.13}$$

$$\mathrm{Res}(+i\beta) = \lim_{p'\to i\beta}(p'-i\beta)\frac{p'^2}{(p'-i\beta)(p'+i\beta)}\frac{1}{\left(p'^2+\alpha^2\right)} \tag{5.14}$$

$$\mathrm{Res}(+i\beta)=\frac{(i\beta)^2}{2i\beta\left(\alpha^2-\beta^2\right)} \tag{5.15}$$

$$\mathrm{Res}(+i\beta)=\frac{i\beta}{2\left(\alpha^2-\beta^2\right)} \tag{5.16}$$

$$\mathrm{Res}(+i\alpha)=\lim_{p'\to i\alpha}(p'-i\alpha)\frac{p'^2}{(p'-i\alpha)(p'+i\alpha)}\frac{1}{\left(p'^2+\beta^2\right)} \tag{5.17}$$

$$\mathrm{Res}(+i\alpha)=-\frac{(i\alpha)^2}{2i\alpha\left(\alpha^2-\beta^2\right)} \tag{5.18}$$

$$\mathrm{Res}(+i\alpha)=-\frac{i\alpha}{2\left(\alpha^2-\beta^2\right)} \tag{5.19}$$

Residue Theorem, $\oint f(z)dz=2\pi i\sum \mathrm{Res}$ (5.20)

$$\int_{-\infty}^{\infty}p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)}\frac{1}{\left(p'^2+\alpha^2\right)}=2\pi i\left(\mathrm{Res}(+i\beta)+\mathrm{Res}(+i\alpha)\right) \tag{5.21}$$

$$\int_{-\infty}^{\infty}p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)}\frac{1}{\left(p'^2+\alpha^2\right)}=2\pi i\left(\frac{i\beta}{2\left(\alpha^2-\beta^2\right)}-\frac{i\alpha}{2\left(\alpha^2-\beta^2\right)}\right) \tag{5.22}$$

$$\int_{-\infty}^{\infty}p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)}\frac{1}{\left(p'^2+\alpha^2\right)}=\frac{\pi(\alpha-\beta)}{\alpha^2-\beta^2} \tag{5.23}$$

$$\int_{-\infty}^{\infty}p'^2dp'\frac{1}{\left(p'^2+\beta^2\right)}\frac{1}{\left(p'^2+\alpha^2\right)}=\frac{\pi}{\alpha+\beta} \tag{5.24}$$

$$C=\lambda\mu C\frac{1}{4\beta}\frac{\partial}{\partial\beta}\frac{\pi}{(\alpha+\beta)} \tag{5.25}$$

$$C=-\frac{\lambda\mu\pi}{4\beta(\alpha+\beta)^2}C \tag{5.26}$$

The intermediate step is

$$1=-\frac{\lambda\mu\pi}{4\beta(\alpha+\beta)^2} \tag{5.27}$$

And then,

$$\alpha=-\beta+\sqrt{-\frac{\lambda\mu\pi}{4\beta}} \tag{5.28}$$

Then we can get the binding energy in fm^{-1} with the help of Eq. (5.6).

3. Results and Discussion

Now we have the analytical result of transition matrix. In this equation the potential strength λ, the form factor parameter β and the incident momentum p_0 are known and the transition matrix element can easily be calculated. From the transition matrix element, we can deduce the phase shift by using the relation $\delta = \frac{\mathrm{Im}(T)}{\mathrm{Re}(T)}$ where Im(*T*) is the imaginary part of transition matrix and Re(T) is real part of transition matrix.

We will discuss the nature of interaction and the phase shift by using the parameter λ=−0.5592 MeV fm^{-1} and β=1.13 fm^{-1} which give the deuteron binding energy. In order to study the nature of interaction, we vary the potential strengths from negative to positive value and produce the phase shifts for various incident energies, which is shown in the figure (1). The nature of phase shift graph is shown in the figure (1).

From the figure (1) we can see that the phase shift is positive for the attractive interaction and negative for the repulsive interaction. From the experiment we can get the scattering cross section and then we can deduce the transition matrix from it. If we know the transition matrix we can calculate the phase shift. From those phase shifts, we can know the nature of interaction whether it is attractive or repulsive.

Here, the known values are β, λ and μ. So, we can calculate α by using Eq. (5.28) in which β = 1.13 fm^{-1}, λ = -0.5592 fm^{-2} and the mass of nucleon is taken as 938.903 MeV, but we are working in momentum space so we have to change MeV to fm^{-1} by dividing ħc = 197.3286 MeVfm. If we have known α, then we can get the binding energy in fm^{-1} with the help of Eq. (5.6) and multiplied by ηc because we usually express the energy in MeV unit.

Now, we may check our program code for numerical results by comparing with the analytical results with Yamaguchi potential. Now The binding energy of deuteron is evaluated analytically and it is 2.22 MeV.

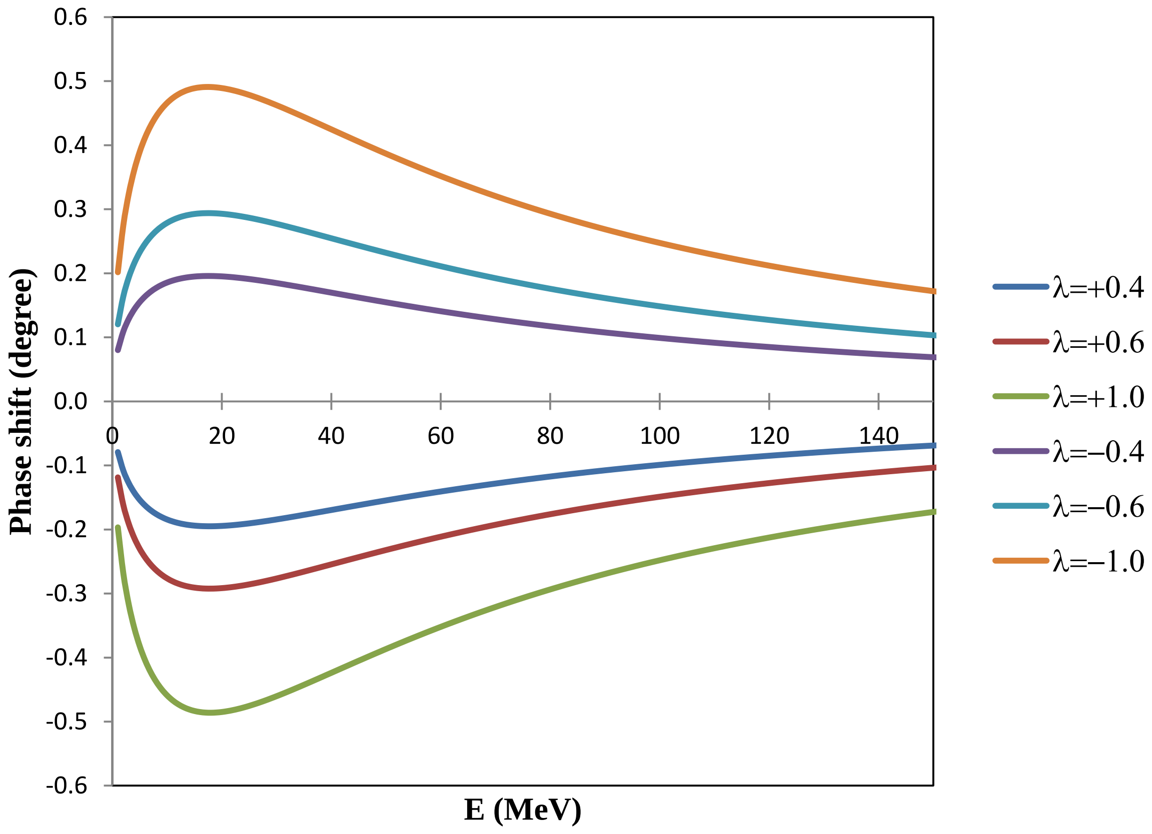

Figure.(1). The phase shift for various incident energies.

4. Conclusion

From the nature of phase shift we can easily determine the nature of interaction whether it is attractive or repulsive of interesting system. Although we have given some knowledge with respect to transition matrix which is very crude, one would be able to continue getting more detailed and advanced facts based on this crude knowledge. It would help the under graduate students to get a better understanding the quantum mechanical scattering process in nuclear particles physics.

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